Fundamental Anomalies

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Introduction

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Introduction

In this paper, we

1. study the q-theory investment CAPM
2. propose a portfolio-independent method
3. estimate model parameters with Bayesian Markov Chain Monte Carlo (MCMC)
4. replicate the size, momentum, profitability, investment, and intangibles premiums
5. miss the value and accruals anomalies

Motivation

Hayashi (1982)

If production and costs are homogenous of degree one in capital and investment:

$$\underbrace{\text{return on investment}}_{\equiv f(X|\theta)} = \underbrace{\text{return on assets}}_{\equiv WACC}$$

where

1. X is the vector of observable firm fundamentals, such as $\mathrm{I/K}$, $\mathrm{Y/K}$
2. θ is the vector of model parameters to be estimated
3. $\text{WACC} = \left(1 - w^B\right) r^{\text{Stock}} + w^B r^{\text{Debt}}$ is observable

Fundamental leveraged stock returns: $f(X|\hat{\theta})$

Research Question

Do Fundamental Returns Exhibit Anomalies?

- Liu, Whited, and Zhang (2009):
  - apply Hayashi (1982) at *portfolio* level
  - GMM target moments: average **portfolio** returns

- Gonçalves, Xue, and Zhang (2020):
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  - GMM target moments: average **portfolio** returns

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  - anomaly construction: parameter estimated for specific **portfolio**
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  - GMM target moments: average **portfolio** returns
  - anomaly construction: parameter estimated for specific **portfolio**

Critique (Campbell (2017)): The parameter values of the model are chosen to fit a specific set of anomalies, and different values are required for different anomalies.

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- construct fundamental firm-level stock returns based on the estimated parameters
- construct fundamental returns of 12 anomalies (potentially ANY anomalies)

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Yes , except the Value and Accruals anomalies

Model and Estimation

Two Capital q-model

Mostly same as in Gonçalves, Xue, and Zhang (2020): Cobb-Douglas production function (parameter γ) with physical and working capital (K and W), plus quadratic adjustment cost of investment in physical capital (parameter a).

Extended for firm i in industry j at $t+1$.

Fundamental Returns

Model-implied fundamental stock return

$$\begin{aligned} r^F_{it+1} &\equiv f(X_{it}, X_{it+1}|\gamma, a) \\ &= \left\{(1-\tau_{t+1})\left[\gamma \left(\frac{Y_{it+1}}{K_{it+1}}\right) + \frac{a}{2}\left(\frac{I_{it+1}}{K_{it+1}}\right)^2\right] + \tau_{t+1}\delta_{it+1}\right. \\ &\quad +(1-\delta_{it+1})\left[1+(1-\tau_{t+1})a\left(\frac{I_{it+1}}{K_{it+1}}\right)\right] \\ &\quad \left.+\frac{W_{it+1}}{K_{it+1}}\right\} \Big/ \left\{(1-w^B_{it})\left[1+(1-\tau_t)a\left(\frac{I_{it}}{K_{it}}\right)+\frac{W_{it+1}}{K_{it+1}}\right]\right\} \\ &\quad -\frac{w^B_{it} r^{Ba}_{it+1}}{1-w^B_{it}}. \end{aligned} \tag{1}$$

Note: shares of capitals γ^K and γ^W are identifiable up to $\gamma \equiv \gamma^K + \gamma^W$.

Fundamental Returns

Model-implied fundamental stock return

$$
\begin{aligned}
r^F_{it+1} \equiv\ & f\left(X_{it}, X_{it+1} | \gamma_{jt+1}, a_{jt+1}, a_{jt}\right) \\
=\ & \Bigg\{ (1-\tau_{t+1}) \left[\gamma_{jt+1} \left(\frac{Y_{it+1}}{K_{it+1}} \right) + \frac{a_{jt+1}}{2} \left(\frac{I_{it+1}}{K_{it+1}} \right)^2 \right] + \tau_{t+1}\delta_{it+1} \\
& + (1-\delta_{it+1}) \left[1 + (1-\tau_{t+1}) a_{jt+1} \left(\frac{I_{it+1}}{K_{it+1}} \right) \right] \\
& + \frac{W_{it+1}}{K_{it+1}} \Bigg\} \Bigg/ \left\{ (1-w^B_{it}) \left[1 + (1-\tau_t) a_{jt} \left(\frac{I_{it}}{K_{it}} \right) + \frac{W_{it+1}}{K_{it+1}} \right] \right\} \\
& - \frac{w^B_{it} r^{Ba}_{it+1}}{1-w^B_{it}}. \qquad (1)
\end{aligned}
$$

Note: shares of capitals γ^K and γ^W are identifiable up to $\gamma \equiv \gamma^K + \gamma^W$.

Models we estimate

We estimate four models to study the time effect and industry effect, along with several models with different adjustment cost functions.

1. $\boldsymbol{\theta}$: constant γ and a
2. $\boldsymbol{\theta}_j$: industry-specific γ_j and a_j
3. $\boldsymbol{\theta}_t$: time-varying γ_t and a_t
4. $\boldsymbol{\theta}_{jt}$: baseline with time-varying and industry-specific γ_{jt} and a_{jt}

Main Results

Outline

Parameter Estimates: Baseline

	Posterior mean	95% CI	Posterior mean	95% CI	
Industry	γ	$[2.5\%, 97.5\%]_\gamma$	a	$[2.5\%, 97.5\%]_a$	$\bar{q}$
Consumer Nondurables	0.13	[0.12, 0.14]	0.42	[0.30, 0.55]	1.05

Parameter Estimates: Baseline

Industry	Posterior mean γ	95% CI $[2.5\%, 97.5\%]_\gamma$	Posterior mean a	95% CI $[2.5\%, 97.5\%]_a$	$\bar{q}$
Consumer Nondurables	0.13	[0.12, 0.14]	0.42	[0.30, 0.55]	1.05
Consumer Durables	0.17	[0.14, 0.19]	1.15	[0.84, 1.48]	1.21
Manufacturing	0.16	[0.15, 0.17]	0.57	[0.52, 0.64]	1.08
Energy	0.20	[0.19, 0.22]	0.45	[0.41, 0.49]	1.06
Business Equipment	0.23	[0.21, 0.25]	1.78	[1.71, 1.86]	1.53
Telecom	0.28	[0.25, 0.31]	0.71	[0.66, 0.77]	1.09
Wholesale & Retail	0.08	[0.08, 0.09]	0.87	[0.79, 0.97]	1.13
Healthcare	0.19	[0.17, 0.21]	0.59	[0.46, 0.74]	1.10
Utilities	0.29	[0.26, 0.33]	0.25	[0.21, 0.32]	1.02
Others	0.17	[0.15, 0.19]	0.48	[0.47, 0.51]	1.09

Value-weighted average marginal q of industry j :

$$\bar{q}_j = \frac{1}{T} \sum_{t=1}^{T} \sum_{i=1}^{N_{jt}} \frac{V_{it-1}}{\sum_{i=1}^{N_{jt-1}} V_{it-1}} \left[1 + a_{jt} \left(1 - \tau_{it}\right) \frac{I_{it}}{K_{it}} \right]$$

Realized vs. Fundamental Firm-Level Returns

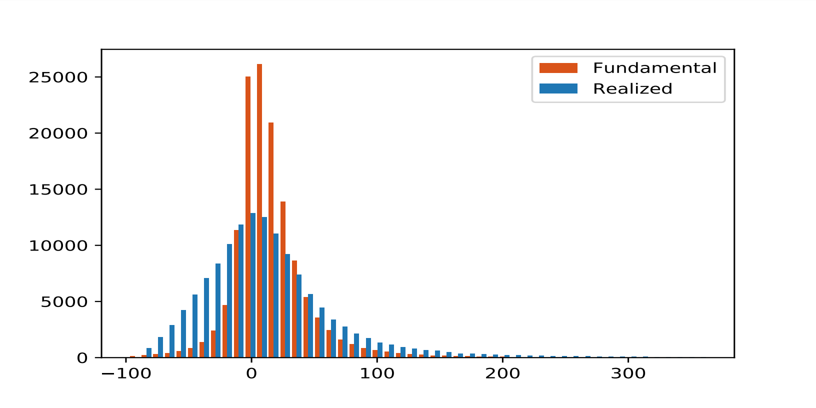


Figure 1: Fundamental returns are less volatile and less skewed, and have thinner tails than the realized returns.

Overall Fit

	Data	$\boldsymbol{\theta}$	$\boldsymbol{\theta}_j$	$\boldsymbol{\theta}_t$	$\boldsymbol{\theta}_{jt}$
Mean	14.45	15.47	15.47	14.97	15.65
		[15.60, 15.83]	[15.36, 15.57]	[14.87, 15.06]	[15.55, 15.75]
StdDev	60.78	19.76	18.49	27.36	34.17
		[19.67, 19.85]	[18.39, 18.59]	[27.26, 27.46]	[34.06, 34.27]
Skewness	2.15	2.12	1.68	1.66	1.68
		[2.11, 2.14]	[1.66, 1.70]	[1.64, 1.67]	[1.67, 1.70]
Kurtosis	11.05	13.33	10.66	10.74	11.20
		[13.26, 13.41]	[10.59, 10.73]	[10.66, 10.82]	[11.11, 11.29]
Correlation	*na*	0.09	0.12	0.12	0.20
		[0.09, 0.10]	[0.12, 0.12]	[0.12, 0.12]	[0.20, 0.20]
m.a.e	*na*	42.45	41.85	40.85	40.10
		[42.42, 42.48]	[41.82, 41.89]	[40.82, 40.88]	[40.06, 40.13]

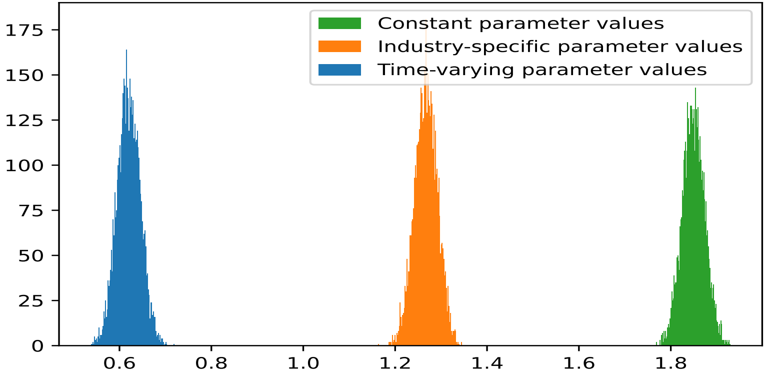


Figure 2: Posterior distributions of the differences in m.a.e. among the four specifications.

Outline

Fundamental anomalies

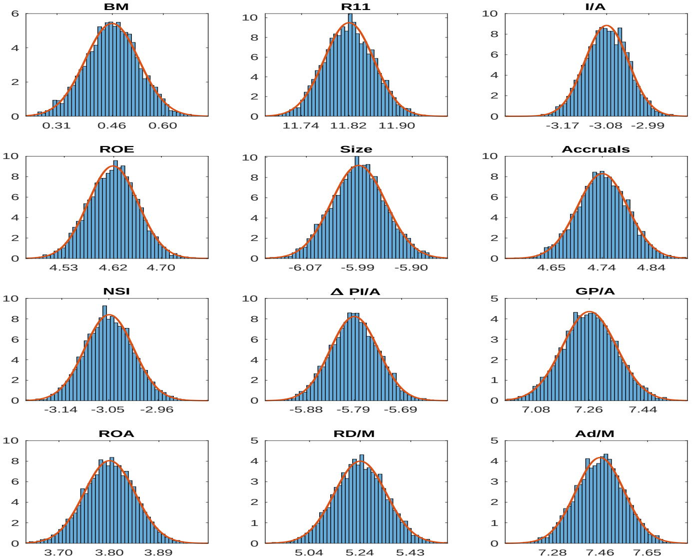
We consider 12 anomalies from six classes as in Hou, Xue, and Zhang (2020):

1. momentum: R11 (Prior 11-month returns)
2. value versus growth: BM
3. investment: I/A (change in total assets), NSI (net stock issues), ΔPI/A (changes in gross property, plant, and equipment plus inventory), Accruals
4. profitability: ROE, ROA, GP/A (total revenue minus cost of goods sold)
5. intangibles: RD/M (R&D expenses), Ad/M (advertising expenses)
6. trading frictions: Size

Fundamental anomalies

Anomaly	r^S	$t(r^S)$	r^F	$t(r^F)$
BM	6.74	2.57	0.46	0.26
			[0.31, 0.60]	[0.18, 0.35]
R11	13.75	4.15	11.82	12.51
			[11.74, 11.90]	[12.38, 12.65]
I/A	-6.30	-3.23	-3.08	-2.25
			[-3.17, -2.99]	[-2.32, -2.18]
ROE	7.69	3.06	4.62	5.72
			[4.53, 4.70]	[5.58, 5.85]
Size	-4.84	-1.37	-5.99	-5.63
			[-6.07, -5.90]	[-5.73, -5.54]
Accruals	-5.58	-3.14	4.74	4.45
			[4.65, 4.84]	[4.34, 4.56]
NSI	-7.65	-4.26	-3.05	-3.36
			[-3.14, -2.96]	[-3.48, -3.25]
ΔPI/A	-5.79	-2.85	-5.79	-4.81
			[-5.88, -5.69]	[-4.93, -4.71]
GP/A	3.87	2.00	7.26	5.84
			[7.08, 7.44]	[5.63, 6.07]
ROA	6.46	2.52	3.80	3.99
			[3.70, 3.89]	[3.86, 4.11]
RD/M	8.70	2.26	5.24	2.12
			[5.04, 5.43]	[2.04, 2.21]
Ad/M	6.10	1.87	7.46	2.82
			[7.28, 7.65]	[2.74, 2.90]

Posterior Distribution of Fundamental Anomalies



Summary of the Main Results

- The fundamental firm-level stock returns closely resemble the realized ones in terms of mean, skewness, and kurtosis and capture over half of the std.
- Time variation and industry variation improves estimation in terms of smaller MAEs.
- The estimated 2-capital model is able to generate significant factor premiums for 10 out of 12 anomalies.

Robustness Analysis

Where the Model Fails

- The value premium:
  - Our estimated adjustment cost parameter is small.
  - The small investment rate is not able to generate large enough return spread.
- The accruals anomaly:
  - The concept of accruals is absent in our model.
  - Cash and accruals basis accountings are treated the same.

Robustness Analysis of Adjustment Cost Functions

- Value premium: asymmetric adjustment costs of investment:
    - $$\Phi(I_{it}, K_{it}) = \frac{a_{jt}^{+}\mathbb{I}_{I_{it}>=0} + a_{jt}^{-}(1 - \mathbb{I}_{I_{it}>=0})}{2}\left(\frac{I_{it}}{K_{it}}\right)^2 K_{it}$$
    - $$\Phi_{it} \equiv \frac{\theta_{jt}}{\nu_{jt}^2}\left[\exp\left(-\nu_{jt}\frac{I_{it}}{K_{it}}\right) + \nu_{jt}\frac{I_{it}}{K_{it}} - 1\right]$$
- Accruals: adjustment cost of working capital
    - $$\Phi(\Delta W_{it}, W_{it}) = \frac{b_{jt}}{2}\left(\frac{\Delta W_{it}}{W_{it}}\right)^2$$

Possible Future Directions

- Value: explicitly modeling intangible capital
- Accruals: explicitly modeling earnings quality

Out-of-Sample Forecasts

Expanding-Window Estimates

	$\alpha \equiv r^S - r^F$	
	Baseline	Expanding-Window Estimates
BM	7.27***	9.63***
	(2.86)	(3.25)
R11	-2.81	6.63
	(-0.71)	(1.69)
I/A	-1.11	-5.27**
	(-0.57)	(-2.21)
ROE	-0.14	0.21
	(-0.05)	(0.06)
Size	2.16	0.88
	(0.56)	(0.23)
Accruals	-8.14***	-7.72***
	(-5.11)	(-4.08)
NSI	-3.87**	-5.31**
	(-2.00)	(-2.00)
ΔPI/A	1.28	-0.73
	(0.85)	(-0.38)
GP/A	-3.78***	-2.49
	(-2.91)	(-1.12)
ROA	-0.74	0.38
	(-0.25)	(0.12)
RD/M	3.38	8.06
	(1.45)	(1.76)
Ad/M	-1.09	1.33
	(-0.33)	(0.37)

Out-of-sample performance

	L	2	3	4	5	6	7	8	9	H	H-L
R-rf	0.36	0.69	0.69	0.76	0.62	0.76	0.72	0.77	0.54	0.80	0.45
	(1.26)	(3.16)	(3.54)	(3.70)	(3.14)	(3.81)	(3.39)	(3.63)	(2.27)	(3.20)	(2.45)
α_{CAPM}	-0.36	0.09	0.10	0.15	0.05	0.17	0.09	0.11	-0.17	0.07	0.43
	(-2.40)	(1.04)	(1.47)	(1.79)	(0.72)	(1.94)	(1.12)	(1.31)	(-1.72)	(0.61)	(2.38)
α_{FF3}	-0.46	0.05	0.11	0.17	0.05	0.19	0.11	0.21	-0.14	0.12	0.58
	(-3.02)	(0.54)	(1.56)	(2.21)	(0.77)	(2.22)	(1.34)	(2.54)	(-1.53)	(1.00)	(3.25)
$\alpha_{Carhart}$	-0.39	0.04	0.11	0.16	0.08	0.17	0.09	0.19	-0.09	0.13	0.52
	(-2.59)	(0.46)	(1.64)	(2.06)	(1.12)	(1.93)	(1.11)	(1.93)	(-0.92)	(1.06)	(2.87)
α_{FF5}	-0.41	0.05	0.04	0.07	-0.07	0.02	0.00	0.11	-0.14	0.20	0.61
	(-2.63)	(0.53)	(0.50)	(0.85)	(-1.09)	(0.28)	(-0.05)	(1.30)	(-1.26)	(1.45)	(3.08)
α_{q4}	0.13	0.40	0.38	0.44	0.25	0.35	0.30	0.50	0.28	0.60	0.47
	(0.87)	(4.02)	(4.98)	(4.54)	(3.27)	(4.01)	(3.18)	(4.76)	(2.46)	(3.99)	(2.22)

- Estimate time-invariant parameters with expanding window
- Predict firm-level fundamentals ($\hat{\boldsymbol{X}}$'s)
- Predict one-month ahead firm-level returns
- Form 10 portfolios based on the predicted returns

Conclusion

Conclusion

1. A novel estimation strategy: estimate a 2-capital model using firm-level stock returns.
2. The model is able to generate economically and statistically significant anomalies in all major categories, except for the value and accrual anomalies.
3. The model shows robust out-of-sample forecast capability, largely due to the imposed economic structure.

Thank You!

Appendix

Economic connotations of parameter estimates

1. γ_{jt}
   γ_{jt} reflects industry j's profit margin as $\Pi_{it} = \gamma_{jt} Y_{it}$.
   1.1 $\overline{\Pi/Y}_{jt} = c_\gamma + b_\gamma \gamma_{jt} + \epsilon^\gamma_{jt}$
   1.2 $\hat{b}_\gamma = 0.13$ (6.10)
2. a_{jt}
   a_{jt} reflects both the marginal costs and marginal benefits of investing one dollar in physical capital and has a positive relation with Tobin's q
   2.1 $\bar{q}_{jt} = c_a + b_a a_{jt} + \epsilon^a_{jt}$
   2.2 $\hat{b}_a = 0.19$ (3.60)

Realized and Fundamental Market Returns

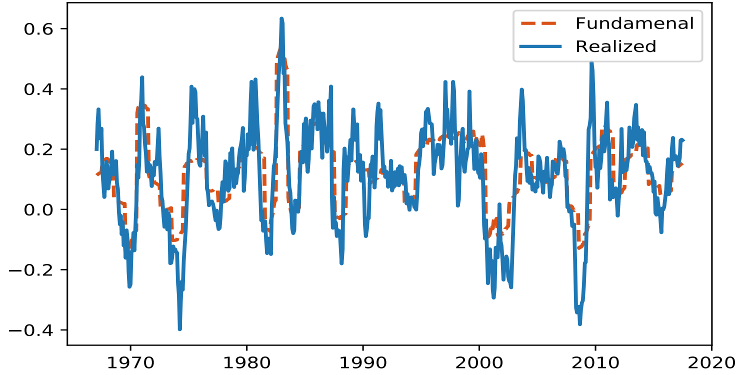
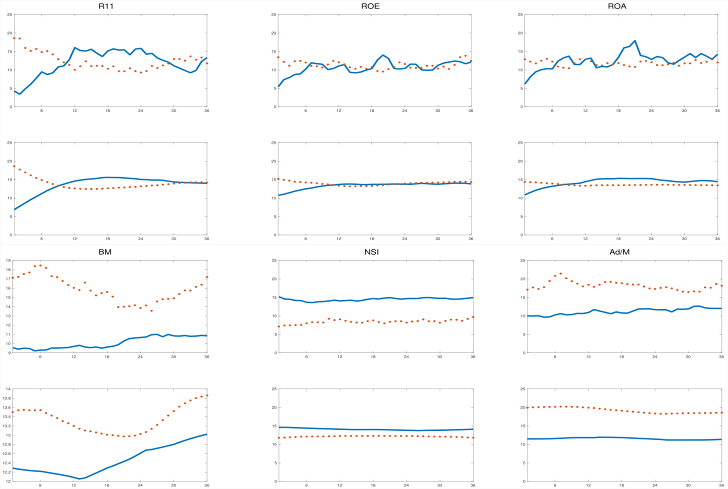
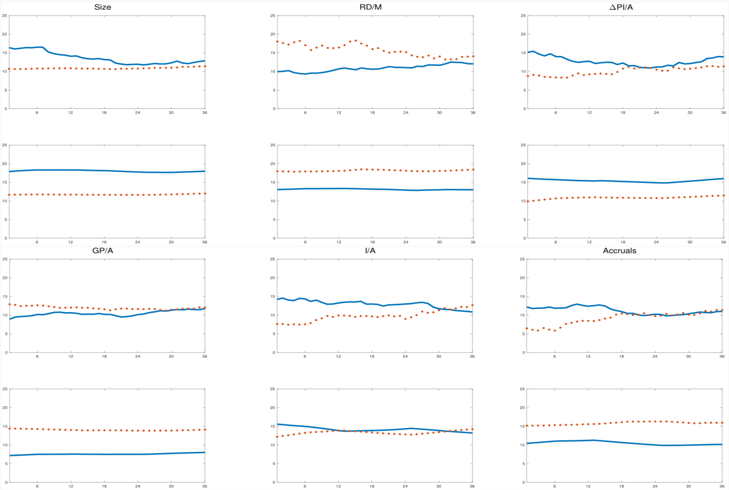


Figure 3: $Corr(R^S_{mt}, R^F_{mt}) = 0.77$

Persistence of factor premiums



Persistence of factor premiums



Market states and factor premiums

	BM		R11		I/A		ROE	
Market State	r^S	r^F	r^S	r^F	r^S	r^F	r^S	r^F
Down	14.24	7.68	-12.99	8.77	-12.97	-3.50	-6.67	1.01
	(5.19)	(1.83)	(-1.03)	(3.05)	(-6.07)	(-1.66)	(-1.31)	(0.39)
Up	5.40	-0.82	18.51	12.43	-5.11	-3.08	10.25	5.29
	(1.81)	(-0.52)	(8.30)	(11.74)	(-2.58)	(-2.08)	(4.35)	(5.09)
	Size		Accruals		NSI		ΔPI/A	
Market State	r^S	r^F	r^S	r^F	r^S	r^F	r^S	r^F
Down	-22.71	-8.31	-7.55	3.15	-4.99	-1.70	-14.60	-9.13
	(-3.40)	(-3.56)	(-2.71)	(1.18)	(-1.09)	(-1.18)	(-3.76)	(-3.27)
Up	-1.66	-5.57	-5.23	5.04	-8.13	-3.34	-4.22	-5.24
	(-0.47)	(-5.09)	(-2.65)	(4.87)	(-4.57)	(-3.49)	(-2.11)	(-4.38)
	GP/A		ROA		RD/M		Ad/M	
Market State	r^S	r^F	r^S	r^F	r^S	r^F	r^S	r^F
Down	-4.66	3.02	-7.19	-2.21	9.35	-1.54	17.17	8.28
	(-2.06)	(0.96)	(-1.04)	(-0.55)	(1.87)	(-0.63)	(4.05)	(1.88)
Up	5.39	8.02	8.89	4.90	8.61	6.12	4.07	7.34
	(2.85)	(6.34)	(3.85)	(4.12)	(1.99)	(2.28)	(1.15)	(2.48)

Comparative statics

	BM	R11	I/A	ROE	Size	Accruals	NSI	$\Delta PI/A$	GP/A	ROA	RD/M	Ad/M
Baseline	6.27	1.87	−3.16	3.05	1.14	−10.33	−4.56	0.04	−3.39	2.63	3.47	−1.39
	(3.33)	(0.57)	(−2.06)	(1.33)	(0.34)	(−6.29)	(−2.90)	(0.03)	(−2.64)	(1.11)	(1.42)	(−0.59)
$\overline{I_{it}/K_{it}}$	13.36	−0.63	−9.88	2.35	1.42	−9.54	−6.91	−5.73	−7.07	1.79	3.90	3.46
	(5.90)	(−0.19)	(−5.47)	(0.96)	(0.40)	(−5.29)	(−3.64)	(−3.92)	(−4.69)	(0.73)	(1.17)	(1.38)
$\overline{I_{it+1}/K_{it+1}}$	−0.12	8.29	0.92	5.36	0.51	−11.67	−3.26	3.75	0.40	4.36	5.13	−5.42
	(−0.06)	(2.24)	(0.55)	(1.93)	(0.15)	(−7.01)	(−2.15)	(2.31)	(0.29)	(1.59)	(2.07)	(−2.16)
$\overline{Y_{it+1}/K_{it+1}}$	−62.26	14.88	5.53	25.52	−14.71	11.17	−36.32	−8.33	109.05	31.19	27.80	6.91
	(−7.47)	(3.69)	(1.72)	(7.62)	(−4.23)	(4.08)	(−5.14)	(−2.36)	(11.25)	(6.67)	(8.17)	(2.54)
$\overline{W_{it+1}/K_{it+1}}$	12.25	1.19	−1.79	2.52	15.83	−19.60	−4.04	9.41	−7.58	3.03	−2.81	−0.44
	(6.03)	(0.32)	(−0.48)	(1.06)	(3.95)	(−7.90)	(−2.00)	(2.48)	(−3.68)	(1.35)	(−0.83)	(−0.13)
θ_j	7.38	10.16	−6.24	3.80	0.98	−10.48	−4.85	−1.58	−2.90	3.40	6.18	−0.29
	(3.25)	(4.06)	(−3.42)	(2.10)	(0.28)	(−5.96)	(−2.44)	(−0.87)	(−1.47)	(1.77)	(1.49)	(−0.10)
θ_t	10.52	8.00	−5.62	2.36	2.73	−14.38	−3.07	−2.67	−11.61	2.35	8.22	−7.79
	(4.20)	(2.93)	(−3.18)	(1.29)	(0.73)	(−7.68)	(−1.49)	(−1.48)	(−5.41)	(1.24)	(2.07)	(−2.42)

- We close the I/K channel by setting it to a constant (cross-sectional median) to find the importance of I/K.
- We close time or industry channels of the parameters to evaluate the importance of parameters.
- We calculate anomaly premiums using the same parameters.

Comparative statics

	BM	R11	I/A	ROE	Size	Accruals	NSI	$\Delta PI/A$	GP/A	ROA	RD/M	Ad/M
Baseline	6.27	1.87	−3.16	3.05	1.14	−10.33	−4.56	0.04	−3.39	2.63	3.47	−1.39
	(3.33)	(0.57)	(−2.06)	(1.33)	(0.34)	(−6.29)	(−2.90)	(0.03)	(−2.64)	(1.11)	(1.42)	(−0.59)
$\overline{I_{it}/K_{it}}$	13.36	−0.63	−9.88	2.35	1.42	−9.54	−6.91	−5.73	−7.07	1.79	3.90	3.46
	(5.90)	(−0.19)	(−5.47)	(0.96)	(0.40)	(−5.29)	(−3.64)	(−3.92)	(−4.69)	(0.73)	(1.17)	(1.38)
$\overline{I_{it+1}/K_{it+1}}$	−0.12	8.29	0.92	5.36	0.51	−11.67	−3.26	3.75	0.40	4.36	5.13	−5.42
	(−0.06)	(2.24)	(0.55)	(1.93)	(0.15)	(−7.01)	(−2.15)	(2.31)	(0.29)	(1.59)	(2.07)	(−2.16)
$\overline{Y_{it+1}/K_{it+1}}$	−62.26	14.88	5.52	25.52	−14.71	11.17	−36.32	−8.33	109.05	31.19	27.80	6.91
	(−7.47)	(3.69)	(1.72)	(7.62)	(−4.23)	(4.08)	(−5.14)	(−2.36)	(11.25)	(6.67)	(8.17)	(2.54)
$\overline{W_{it+1}/K_{it+1}}$	12.25	1.19	−1.79	2.52	15.83	−19.60	−4.04	9.41	−7.58	3.03	−2.81	−0.44
	(6.03)	(0.32)	(−0.48)	(1.06)	(3.95)	(−7.90)	(−2.00)	(2.48)	(−3.68)	(1.35)	(−0.83)	(−0.13)
θ_j	7.38	10.16	−6.24	3.80	0.98	−10.48	−4.85	−1.58	−2.90	3.40	6.18	−0.29
	(3.25)	(4.06)	(−3.42)	(2.10)	(0.28)	(−5.96)	(−2.44)	(−0.87)	(−1.47)	(1.77)	(1.49)	(−0.10)
θ_t	10.52	8.00	−5.62	2.36	2.73	−14.38	−3.07	−2.67	−11.61	2.35	8.22	−7.79
	(4.20)	(2.93)	(−3.18)	(1.29)	(0.73)	(−7.68)	(−1.49)	(−1.48)	(−5.41)	(1.24)	(2.07)	(−2.42)

- We close the I/K channel by setting it to a constant (cross-sectional median) to find the importance of I/K.
- We close time or industry channels of the parameters to evaluate the importance of parameters.
- We calculate anomaly premiums using the same parameters.

Bayesian MCMC

For firm i in industry j (Fama-French 10 industries):

$$\underbrace{r^S_{it+1}}_{\text{realized return}} = \underbrace{f(X_{it}, X_{it+1} | \gamma_{jt+1}, a_{jt+1}, a_{jt})}_{\text{fundamental return: } r^F_{it+1}} + \underbrace{\varpi_{it}^{-1/2} \sigma_r e^r_{it+1}}_{\substack{\text{estimation error} \\ e^r \sim \mathcal{N}(0,1) \\ \varpi_{it} \equiv \frac{V_{it}}{\sum_{i=1}^{N_t} V_{it}}}}$$

- Assumption:
$$\begin{aligned} \gamma_{jt+1} &= \gamma_{jt} + \sigma_\gamma e^\gamma_{jt+1}, e^\gamma \sim \mathcal{N}(0,1) \\ a_{jt+1} &= a_{jt} + \sigma_a e^a_{jt+1}, e^a \sim \mathcal{N}(0,1) \\ e^\gamma &\perp e^a \end{aligned}$$

Bayesian MCMC

- Four specifications: use $\boldsymbol{\theta}$ to denote a generic parameter vector.
  - constant parameters ($\boldsymbol{\theta}$)
  - parameters with industry variations ($\boldsymbol{\theta}_j$)
  - parameters with time variations ($\boldsymbol{\theta}_t$)
  - parameters with industry and time variations ($\boldsymbol{\theta}_{jt}$)

Bayesian MCMC with Gibbs Sampling

Target

Obtain samples from a joint distribution.

Procedure

Observed data $\{\boldsymbol{X}\}$

Set initial values $\gamma_{jt+1}^{(0)}, a_{jt+1}^{(0)}$ for all j and t

for $g = 1$ *to* G **do**

  for $j = 1$ *to* J, $t = 1$ *to* T **do**

    $\gamma_{jt+1}^{(g)} = f(\gamma_{jt+1} | \boldsymbol{X}, \boldsymbol{\theta}^{\max\{g,g-1\}})$.

    $a_{jt+1}^{(g)} = f(a_{jt+1} | \boldsymbol{X}, \boldsymbol{\theta}^{\max\{g,g-1\}})$.

  end

end

end

▷ We use Metropolis-Hastings algorithm embedded Gibbs sampling.

Bayesian MCMC with Gibbs Sampling

Target

Obtain samples from a high dimensional joint posterior distribution.

Procedure

  Observed data $\{\boldsymbol{X}\}$

  Set initial values $\gamma_{jt+1}^{(0)}, a_{jt+1}^{(0)}$ for all j and t

  for $g = 1$ *to* G **do**

    for $j = 1$ *to* J, $t = 1$ *to* T **do**

      $\gamma_{jt+1}^{(g)} = f(\gamma_{jt+1}|\boldsymbol{X}, \boldsymbol{\theta}^{\max\{g,g-1\}})$.

      $a_{jt+1}^{(g)} = f(a_{jt+1}|\boldsymbol{X}, \boldsymbol{\theta}^{\max\{g,g-1\}})$.

    end

  end

end

▷ We use Metropolis-Hastings algorithm embedded Gibbs sampling.

Bayesian MCMC with Gibbs Sampling

Toy example

- prior belief $\pi(\boldsymbol{\theta})$
- conditional likelihood $f(\boldsymbol{r}|\boldsymbol{\theta})$
- posterior $f(\boldsymbol{\theta}|\boldsymbol{r}) \propto \pi(\boldsymbol{\theta}) f(\boldsymbol{r}|\boldsymbol{\theta})$

$$\theta_1^{(0)}, \theta_2^{(0)}, \cdots, \theta_d^{(0)}$$

Bayesian MCMC with Gibbs Sampling

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Bayesian MCMC with Gibbs Sampling

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$$\theta_1^{(1)}, \theta_2^{(1)}, \cdots, \theta_d^{(1)}$$

- Bayesian MCMC is an iterative sampling method
- Hammersley–Clifford theorem guarantees the MCMC draws from conditional marginals converge to the joint
- We use the posterior mean as the final parameter estimation

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